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3-PRIME CORDIAL LABELING OF SOME CYCLE RELATED SPECIAL GRAPHS

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ABSTRACT

Let G be a (p,q) graph. Let $f : V(G) \rightarrow \{1,2,\ldots k\}$ be a function. For each edge *uv*, assign the label gcd (f(u),f(v)). F is called k-prime cordial labeling of G if $| v_f(i) - v_f(j) | \leq 1$, $i,j \in \{1,2,\ldots k\}$, and $| e_f(0) - e_f(1) | \leq 1$ where $v_f(x)$ denotes the number of vertices labeled with x, $e_f(1)$ and $e_f(0)$ respectively the number of edges labeled with 1 and not labeled with 1. A graph which admits a k-prime cordial labeling is called a k-prime cordial graph. In this paper, we investigate the 3-prime cordial labeling behaviour of some triangular snake graphs and diamond snake graphs..

Keywords: *labeling, prime cordial labeling, dragon graphs, armed crown graphs, dumbbell graphs and jewel graphs.*

I. INTRODUCTION

A graph labeling is an assignment of integers to the vertices or edges or both subject to some conditions. Labeled graphs are becoming an increasingly useful family of Mathematical Models from a broad range of applications. The graph labeling problem has a fast development recently. This problem was first introduced by Alex Rosa [5] in 1967. Since Rosa's article, many different types of graph labeling problems have been defined around this. This is not only due to its mathematical importance but also because of the wide range of the applications arising from this area, for instance, x-rays, crystallography, coding theory, radar, astronomy, circuit design, and design of good radar type codes, missile guidance codes [2]. All graphs considered here are finite simple and undirected. Throughout this paper we have considered only simple and undirected graph. Terms and definitions not defined here are used in the sense of Harary [3] and Gallian [2]. Let G=(V,E) be a (p,q) graph. Cahit [1] initiated the concept of cordial labeling of graphs. Sundaram et al. [6] introduced the concept of prime cordial labeling of graphs. Motivated by the above labeling, the notion of k-prime cordial labeling has been introduced by Ponraj et al. [4] and they studied the k-prime cordial labeling behaviour of paths, cycles, and bistars of even order. Also they studied about the 3-prime cordiality of paths, cycles, corona of tree with a vertex, comb, crown, olive tree and some more graphs [9]. In this paper, we investigate the 3-prime cordial labeling behaviour of some dragon graphs, armed crown graphs, dumbbell graphs and jewel graphs.

II. PRELIMINARY RESULTS

Definition 2.1. A graph labeling is the assignment of unique identifiers to the edges and vertices of a graph.

Definition 2.2. Let G be a (p,q) graph. Let $f : V(G) \rightarrow \{1,2,\ldots k\}$ be a function. For each edge *uv*, assign the label gcd (f(u),f(v)). f is called k-prime cordial labeling of G if $| v_f(i) - v_f(j) | \leq 1$, $i,j \in \{1,2,\ldots k\}$, and $| e_f(0) - e_f(1) | \leq 1$ where $v_f(x)$ denotes the number of vertices labeled with x, $e_f(1)$ and $e_f(0)$ respectively the number of edges labeled with 1 and not labeled with 1. A graph which admits a k-prime cordial labeling is called a k-prime cordial graph.

Theorem 2.2. [10] The path Pn is 3-prime cordial if and only if $n \neq 3$

Theorem 2.3. [10] Dragon $C_m@P_n$ is obtained from the cycle C_m : $u_1u_2\ldots u_m\ v_1$ and the path P_n $v_1v_2\ldots v_n$ by identifying u_m with v_1

Theorem 2.4. [10] The Jewel graph J_n is a graph with vertex set $V(J_n) = \{u, x, v, y, u_i : 1\le i \le n\}$ and edge set $E(J_n) = \{ux, xv, vy, uy, xy, uu_i, vu_i : 1 \le i \le n\}$

Theorem 2.5. [10] The armed crown graph AC_n is a graph in which a P_m is attached with each vertex of cycle C_n

Theorem 2.6. [10] The graph obtained by joining two disjoint cycles $u_1u_2\ldots u_nu_1$ and $v_1v_2\ldots v_nv_1$ with an edge u_1v_1 is called dumbbell graph Db_n

III. MAIN RESULTS

Theorem 3.1. The armed crown graph AC_n is 3-Prime cordial.

Proof:

Let $u_1\ u_2\ldots\ u_nu_1$ be a cycle C_n. Let $V(AC_n) = V(C_n) \cup \{v_i\ w_i : 1 \le i \le n\}$
and $E(AC_n) = E(C_n) \cup \{u_iv_i, u_iw_i : 1 \le i \le n\}$
Clearly order and size of AC_n are 3n and 3n respectively.

Case (i) $n \equiv 0 \pmod 6$ Let $n = 6t$, $t \ge 1$
Assign the label 2 to the vertices u_i, v_i, w_i $(1 \le i \le 2t)$ and assign the labels 3 to the vertices u_i, v_i, w_i $(2t+1 \le i \le t)$. Next assign the label 3 to the vertices u_{3t+1}, v_{3t+1}. Next assign the label 3 to the vertices $w_{3t+2}, w_{3t+3}\ldots w_{6t+1}$. Finally assign the label 1 to the remaining non-labelled vertices.

Case (ii) $n \equiv 1 \pmod 6$ Let $n = 6t+1$
Assign the label to the vertices u_i, v_i, w_i $(1 \le i \le n-1)$ as in case (i). Next assign the labels 2,1,3 respectively to the vertices u_n, v_n, w_n. Finally interchange the labels of w_{3t+1} and u_{3t+2}.

Case (iii) $n \equiv 2 \pmod 6$ Let $n = 6t+2$
As in case (ii) assign the labels to the vertex u_i, v_i, w_i $(1 \le i \le n-1)$. Finally, assign the labels 2, 1, 3 to the vertices u_n, v_n, w_n respectively.

Case (iv) $n \equiv 3 \pmod 6$ Let $n = 6t+3$
In this case, assign the labels to the vertices u_i, v_i, w_i $(1 \le i \le n-1)$ as in case (iii). Next, assign the labels 2, 1, 3 respectively to the vertices u_n, v_n, w_n.

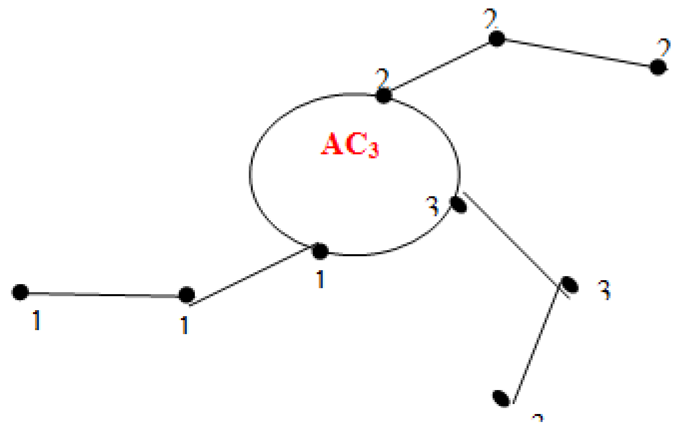
Case (v) $n \equiv 4 \pmod 6$ Let $= 6t+4$.
As in case (iv) assign the labels to the vertices u_i, v_i,w_i $(1 \le i \le n-1)$. Next, assign the labels 2, 1, 3 respectively to the remaining non-labelled vertices. Finally interchange the labels of u_{3t+3} and w_{3t+3}.

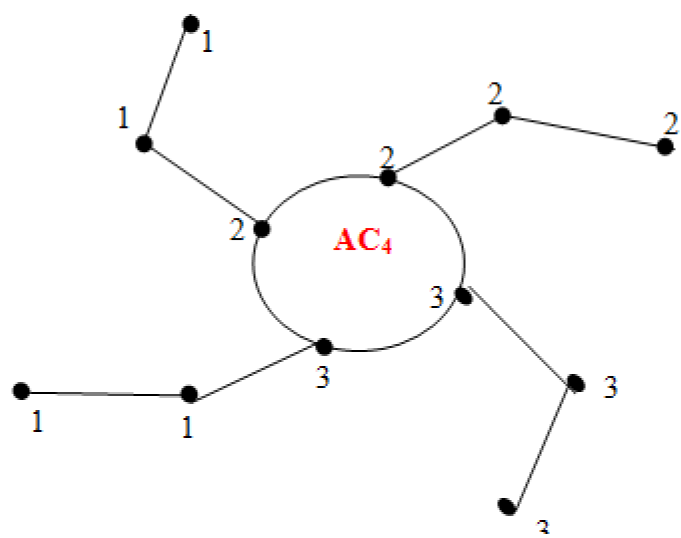
Case (vi)
$n \equiv 5 \pmod 6$ Let $n = 6t+5$. Using the same technique, assign the labels to the vertices u_i, v_i, w_i $(1 \le i\le n-1)$ as in case (v). Finally assign the labels 2, 1, 3 respecting to the vertices u_n, v_n and w_n. The Table 1 establishes that this vertices labeling is a 3-Prime cordial labeling of AC_n , for $n \ge 6$

Table 1

n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$e_f(1)$	$e_f(0)$
6t	2t	2t	2t	9t	9t
6t+1	2t+1	2t+1	2t+1	9t+1	9t+2
6t+2	2t+2	2t+2	2t+2	9t+3	9t+3

6t+3	2t+2	2t+2	2t+2	9t+5	9t+4
6t+4	2t+2	2t+2	2t+2	9t+6	9t+6
6t+5	2t+2	2t+2	2t+2	9t+8	9t+7





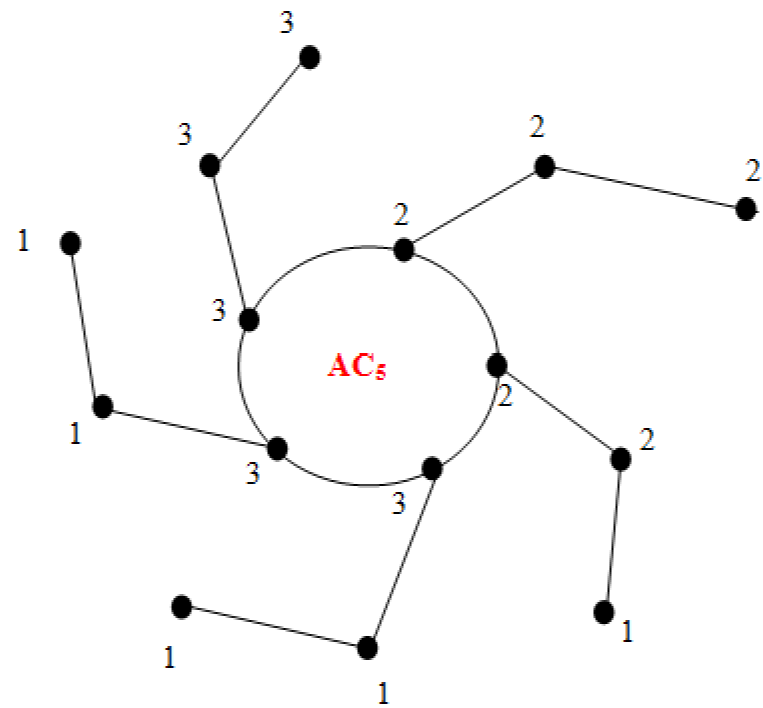


Figure 1 The 3-Prime cordial labeling of AC_3, AC_4 and AC_5 are shown in Figure 1

Theorem: 3.2: The dumbbell graph $\mathbf{Db_n}$ is 3-Prime cordial.
Proof:
The dumb bell graph $\mathbf{Db_n}$ is obtained by joining two disjoint cycles $u_1 u_2 \ldots u_n u_1$ and $v_1 v_2 \ldots v_n v_1$ of same length with an edge u_1v_1. Clearly, $\mathbf{Db_n}$ has 2n vertices and 2n+1 edges.

Case (i) $n \equiv 0 \pmod 6$ Let $n = 6t$
Assign the label 2 to the vertices $u_1, u_2 \ldots u_{4t}$ and 3 to the vertices $u_{4t+1}, u_{4t+2} \ldots u_n$. We now consider the next cycle. Assign the label 3 to the vertices v_1, v_2, v_3. We now assign the label 3 to the vertices $v_5, v_7, v_9, \ldots v_{4t-3}$. Finally, assign the label 1 to the remaining non-labelled vertices.

Case (ii) $n \equiv 1 \pmod 6$ Let $n = 6t+1$.
Assign the label to the vertices u_i, v_i $(1 \le i \le n-1)$ as in case (i). Next, assign the labels 3, 1 respectively to the vertices u_n and v_n.

Case (iii) $n \equiv 2 \pmod 6$ Let $n = 6t+2$
In this case, assign the label to the vertices u_i, v_i, $(1 \le i \le n-1)$ as in case (ii). Finally, assign the label 2, 1 to the vertices u_n and v_n respectively.

Case (iv) $n \equiv 3 \pmod 6$ Let $n = 6t+3$
As in case (iii) assign the label to the vertex u_i, v_i $(1 \le i \le n-1)$. Next, assign the labels 2, 3 respectively to the vertices u_n and v_n.

Case (v) $n \equiv 4 \pmod 6$ Let $n = 6t+4$
In this case, assign the label to the vertices u_i, v_i $(1 \le i \le n-1)$ as in case (iv). Finally, assign the labels 2, 1 to the vertices u_n and v_n respectively.

Case (vi) $n \equiv 5$ Let $n = 6t+5$

As in case (v), assign the label to the vertices u_i, v_i $(1 \le i \le n\text{-}1)$. Next, assign the label 1, 3 respectively to the vertices u_n, v_n. Finally interchange the labels of v_4 and v_5.

This vertex labelling is a 3-prime cordial labelling, follows from the Table 2.

Table 2

n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$e_f(1)$	$e_f(0)$
6t	4t	4t	4t	6t	6t+1
6t+1	4t+1	4t	4t+1	6t+1	6t+2
6t+2	4t+2	4t+1	4t+1	6t+2	6t+3
6t+3	4t+2	4t+2	4t+2	6t+3	6t+4
6t+4	4t+3	4t+3	4t+2	6t+4	6t+5
6t+5	4t+4	4t+3	4t+3	6t+5	6t+6

Theorem:3.3: The dragon graph $\mathbf{C_m@\ P_n}$ is 3-Prime cordial for all values of $m \ge 3$ and $n \ge 3$.

Proof:

Let $\mathbf{C_m}$: $u_1u_2\ldots.. u_mv_1$ and $\mathbf{P_n}$: $v_1v_2\ldots.v_n.$ The path P_n is 3-Prime cordial [ ]. We now give a 3-Prime cordial labelling of the path P_n.

Case (i) $n \equiv 0 \pmod 6$

Let n = 6t. Assign the label 2 to the first 2t vertices $v_1, v_2\ldots..v_{2t}$ and 3 to the next vertices $v_{2t+1}, v_{2t+2} \ldots\ldots v_{3t}$ and v_{3t+1}. Next, assign the label 3 to the vertices $v_{3t+3}, v_{3t+5}\ldots\ldots$, and v_{5t-1}. Finally, assign the label 1 to the remaining non-labelled vertices.

Case (ii) $n \equiv 1 \pmod 6$

Let n = 6t+1. As in case (1) assign the label to v_i $(1 \le i \le n\text{-}1)$. Next, assign the label 1 to the vertex v_n.

Case (iii) $n \equiv 2 \pmod 6$

Let n = 6t+2. Assign the label to the vertices v_i $(1 \le i \le n\text{-}1)$ as in case (i). Now, assign the label 2 to the vertex v_n. Finally, interchange the labels of v_{3t+2} and v_{3t+3}.

Case (iv) $n \equiv 3 \pmod 6$

Let n = 6t+2. As in case (i), assign the label to the vertices v_i $(1 \le i \le n\text{-}1)$. Next assign the label 3 to the vertex v_n.

Case (v) $n \equiv 4 \pmod 6$

Let n = 6t+4. In this case, as in previous case (iv), assign the label to the vertices v_i $(1 \le i \le n\text{-}1)$ and assign the label 3 to the last vertex v_n.

Case (vi) $n \equiv 5 \pmod 6$

Let n = 6t+5. As in case (v) assign the label to the vertices v_i $(1 \le i \le n\text{-}1)$. Next assign the label 2 to the vertex v_n and interchange the labels of v_n and v_{n-3}.

Define $h = V(\mathbf{C_m@P_n}) \rightarrow \{1,2,3\}$ by $\mathbf{h}(u_i) = \mathbf{f}(u_i) : 1 \le i \le m$

and $\mathbf{h}(v_i) = \mathbf{f}(v_i)$: $1 \le i \le n$

Case (i) $m \equiv 0 \pmod 6$, $n \equiv 0 \pmod 6$ In this case,

$$\mathbf{v}_h(1) = \frac{m}{3} + \frac{n}{3}$$

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$\mathbf{v}_h\,(2) \quad = \dfrac{m}{3} + \dfrac{n}{3}$

$\mathbf{v}_h\,(3) \quad = \dfrac{m}{3} + \dfrac{n}{3}$

Also, $\mathbf{e}_h(1) \quad = \dfrac{m}{2} + \dfrac{n}{2} - 1$

Case (ii) $m \equiv 0 \pmod 6$, $n \equiv 1 \pmod 6$
Let $m = 6t_1$, $n = 6t_2+1$ Here,
$\mathbf{v}_h\,(1) = 2t_1 + 2t_2+1$ $\quad$ $\mathbf{v}_h\,(2) = v_n(3) = 2t_1+2t_2$
$\mathbf{e}_h\,(0) = 3t_1 + 3t_2$ $\quad$ $\mathbf{e}_h\,(1) = 3t_1 + 3t_2$

Case (iii) $m \equiv 0 \pmod 6$, $n \equiv 2 \pmod 6$
Let $m = 6t_1$, $n = 6t_2+2$ In this case,
$\mathbf{v}_h\,(1) = \mathbf{v}_h\,(2) = 2t_1+2t_2+1$
$\mathbf{v}_h\,(3) = 2t_1 + 2t_2$
$\mathbf{e}_h\,(0) = 3t_1 + 3t_2 + 1$
$\mathbf{e}_h\,(1) = 3t_1+3t_2$

Case (iv) $m \equiv 1 \pmod 6$, $n \equiv 0 \pmod 6$
Let $m = 6t_1+1$, $n = 6t_2$
$\mathbf{v}_h\,(1) = 2t_1+2t_2+1$
$\mathbf{v}_h\,(2) = v_n(3) = 2t_1+2t_2$
$\mathbf{e}_h\,(1) = 3t_1+3t_2$ $\quad$ $\mathbf{e}_h\,(0) = 3t_1+3t_2-1$

Case (v)
$m \equiv 1 \pmod 6$, $n \equiv 1 \pmod 6$
Re-label the vertices v_i with $f(v_i) = 1$ and i is small by 2. Clearly this vertex labelling is a 3-Prime cordial labelling. Proceeding likes this we can re-label the vertices if needed and we get a 3-Prime cordial labelling. as shown in Figure 2 for $C_7@P_5$

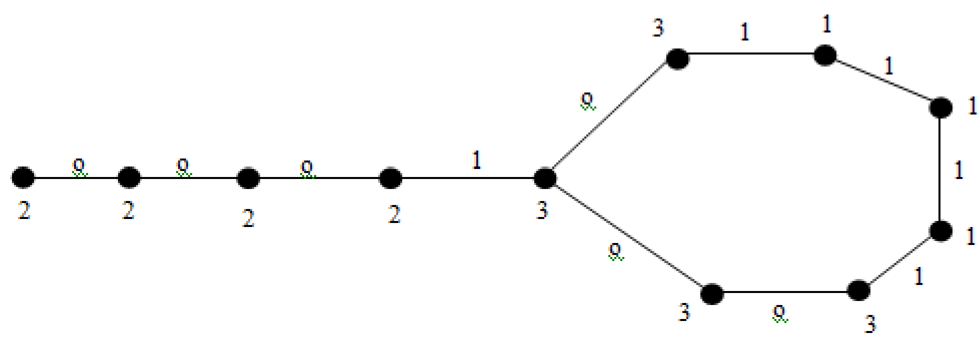

Figure 2 3-Prime cordial Labelng Dragon Graph $C_7@P_5$

Theorem:3.4: The Jewel graph $\mathbf{J_n}$ is not 3-Prime cordial for all values of n.
Proof: Let $\quad V(J_n) = \{u, x, v, y, u_i : 1 \le i \le n\}$
$\quad E(J_n) = \{ux, xv, vy, uy, xy, uu_i, vu_i : 1 \le i \le n\}$
Suppose $\mathbf{J_n}$ is a 3-Prime cordial graph with a 3- Prime cordial labeling **f**

Case (i) **f**(u) = 1 and **f**(v) = 1.In this case, all the edges uu_i, vu_i $(1 \le i \le n)$ receive the label 1. That is $e_f(1) \ge 2n$, a contradiction.

Case (ii) **f**(u) = 1 and **f**(u) = 2. The edge uu_i $(1 \le i \le n)$ receives the label 1 and the edges vu_i with **f**(u_i) = 3 receives the label 1. This implies $|e_f(0) – e_f(1)| >1$, a contradiction

Case (iii) **f**(u) = 1 and **f**(v) = 3

In this case the edges uu_i $(1 \le i \le n)$ and the edges vu_i with **f**(u_i) = 2 receives the label 1. Here also, $|e_f(0) – e_f(1)| > 1$, a contradiction. Thus $\mathbf{J_n}$ is not 3-Prime cordial for all values of n.

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